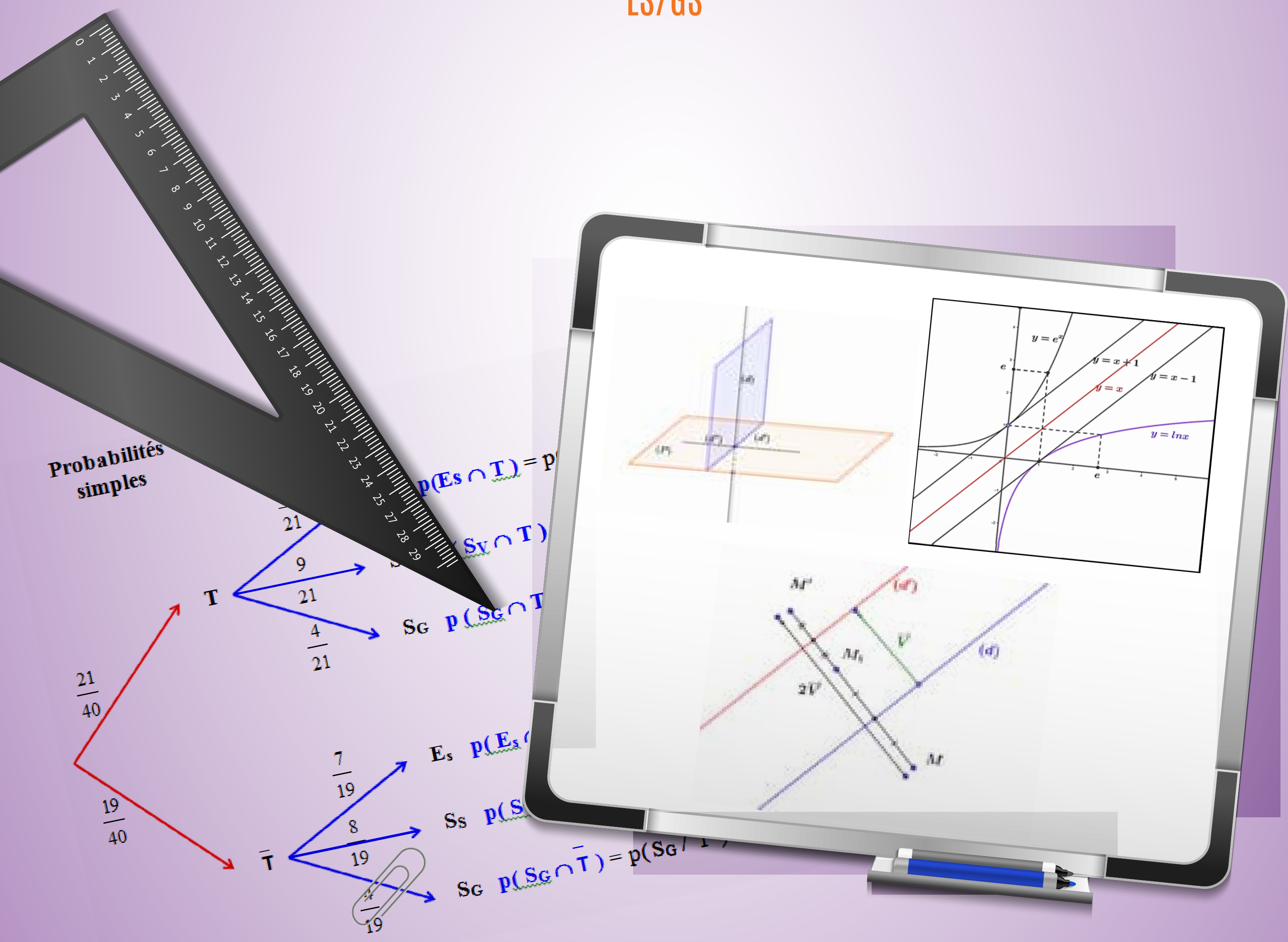
Edition 2021

REVISION & PRACTICE

Mathematics

LS/GS



Paul JALWAN
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DAR ALMOUFID Publishers

Dear students,

We have come across a large number of requests from those who wish to have access to abstract concepts, training exercises and problems, so they can prepare themselves for the Lebanese Baccalaureate and university entrance exams.

As a response, we, university and third secondary teachers of **Mathematics**, **Physics**, **Chemistry** and **Biology**, took the initiative to put at hand the adequate tools and specific activities required by the revised Lebanese Curricula.

We emphasize on the quality of training and explanations to facilitate the comprehension of concepts and acquire reasoning and problem-solving skills.

The importance of "**Revision and Practice**" lies in tackling "online" the reduced program, through the right methodology for the acquisition of knowledge and performance.

By collaborating with you, with Dar Al Moufid, with your parents and with the academic institutions, we hope we will provide you with the tools to overcome difficulties and reach your educational goals, despite the current national socio-economic situation.

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Content

2- Vertical and horizontal shifts: (*fig.14*)

suppose that $c > 0$:

- $y = f(x) + c$, we shift the graph of $y = f(x)$ a distance c upward.
- $y = f(x) - c$, we shift the graph of $y = f(x)$ a distance c downward.
- $y = f(x - c)$, we shift the graph of $y = f(x)$ c units to the right.
- $y = f(x + c)$, we shift the graph of $y = f(x)$ c units to the left.

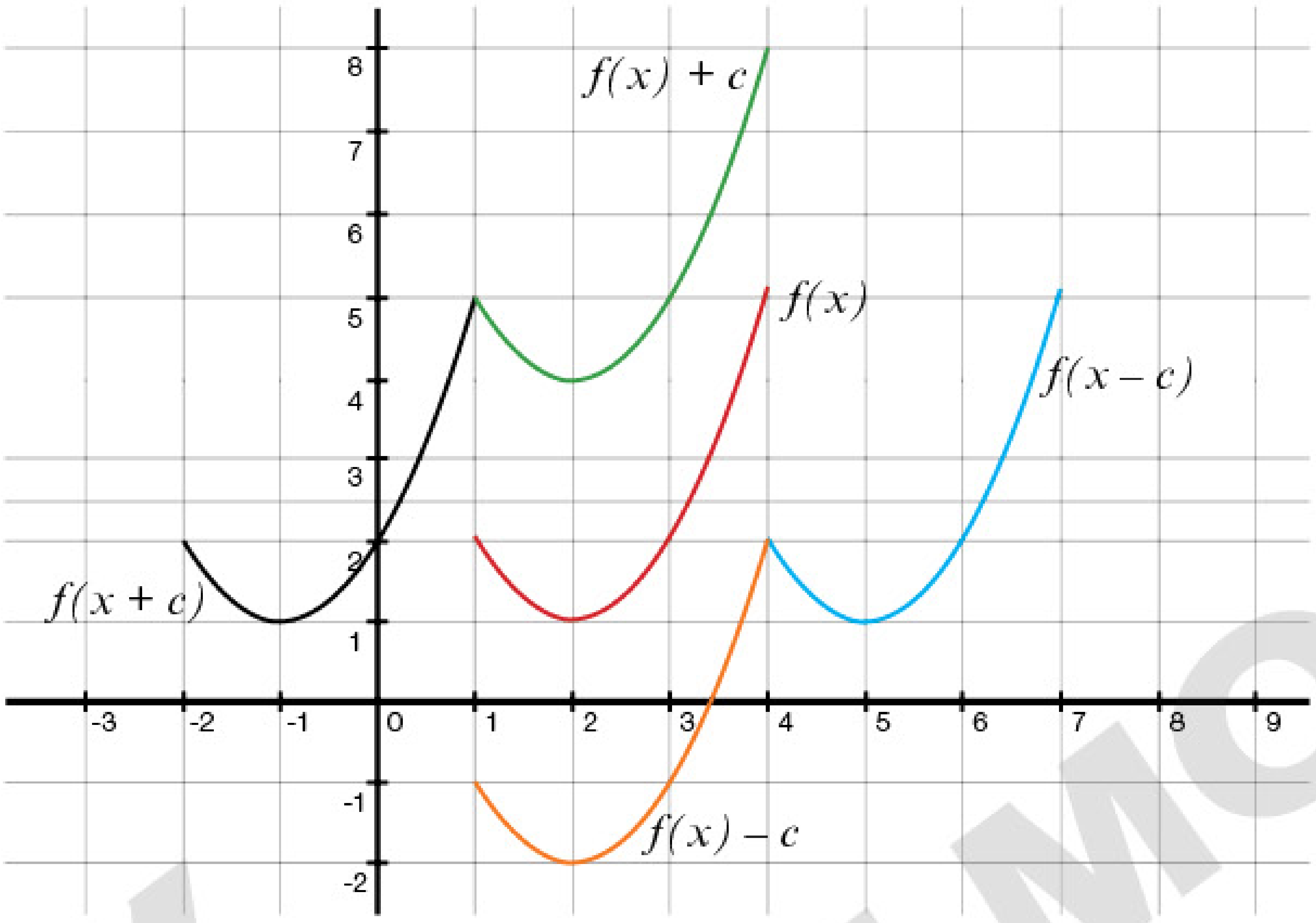

fig.14

3- Sketch the graph of function:

$y = f(x) = | x^2 - 1 |$ (*fig.15*)

The graph of $f(x) = x^2 - 1$ is a shifted graph of $y = x^2$ downward by 1 unit.

For $x \in]-1\ ;\ 1[$; $f(x) = y = -x^2 + 1$ as $(x^2 - 1)$ is negative. $y_1 = -y = 1 - x^2$;

y_1 is symmetric of y about the x-axis.

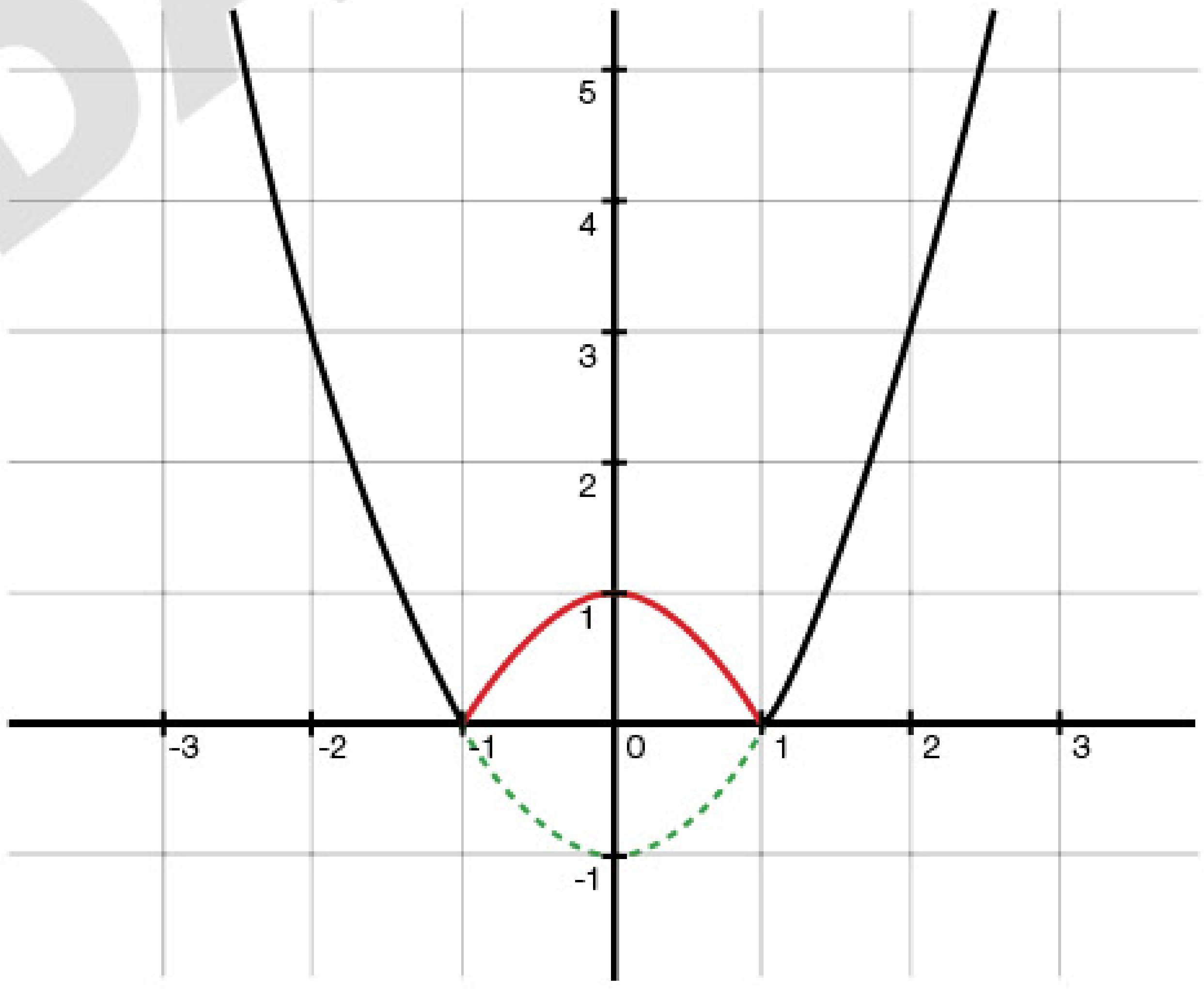

fig.15

XVIII- Exponential function

If $y = ln\,x$, then $x = e^y$, by changing $x \Longleftrightarrow y$; $y = e^x$, is called the **exponential function.**
$E\,(x\;,\,y)$ belongs to the curve of : $y = ln\,x$
$F\,(y\;,\,x)$ belongs to the curve of $y = e^x$, by changing x into y
Then E and F are symetric about $y = x$, the first bisector of the plan (*fig.37*).

We can conclude that: $\lim\limits_{x \to +\infty} e^x \to +\infty$ and $\lim\limits_{x \to -\infty} e^x \to 0^+$

1- Properties of $f(x) = e^x$

For any numbers a and b:

- $e^{a+b} = e^a \,.\, e^b$ since: $ln\, e^{a+b} = a + b = ln\, e^a + ln\, e^b = ln\, e^a \,.\, e^b$
- $e^{a-b} = e^{a+(-b)} = e^a \,.\, e^{(-b)} = \dfrac{e^a}{e^b}$
- $(e^a)^b = e^{ab}$, as in $ln\,(e^a)^b = b\; ln\, e^a = ba = ln\, e^{ba}$

  for any $x \in \mathbb{R}$, $ln e^x = x$. Let $y = e^x$ there is for:

  $(lny)' = (x)'$; $\dfrac{y'}{y} = 1$ and $y' = y = e^x$

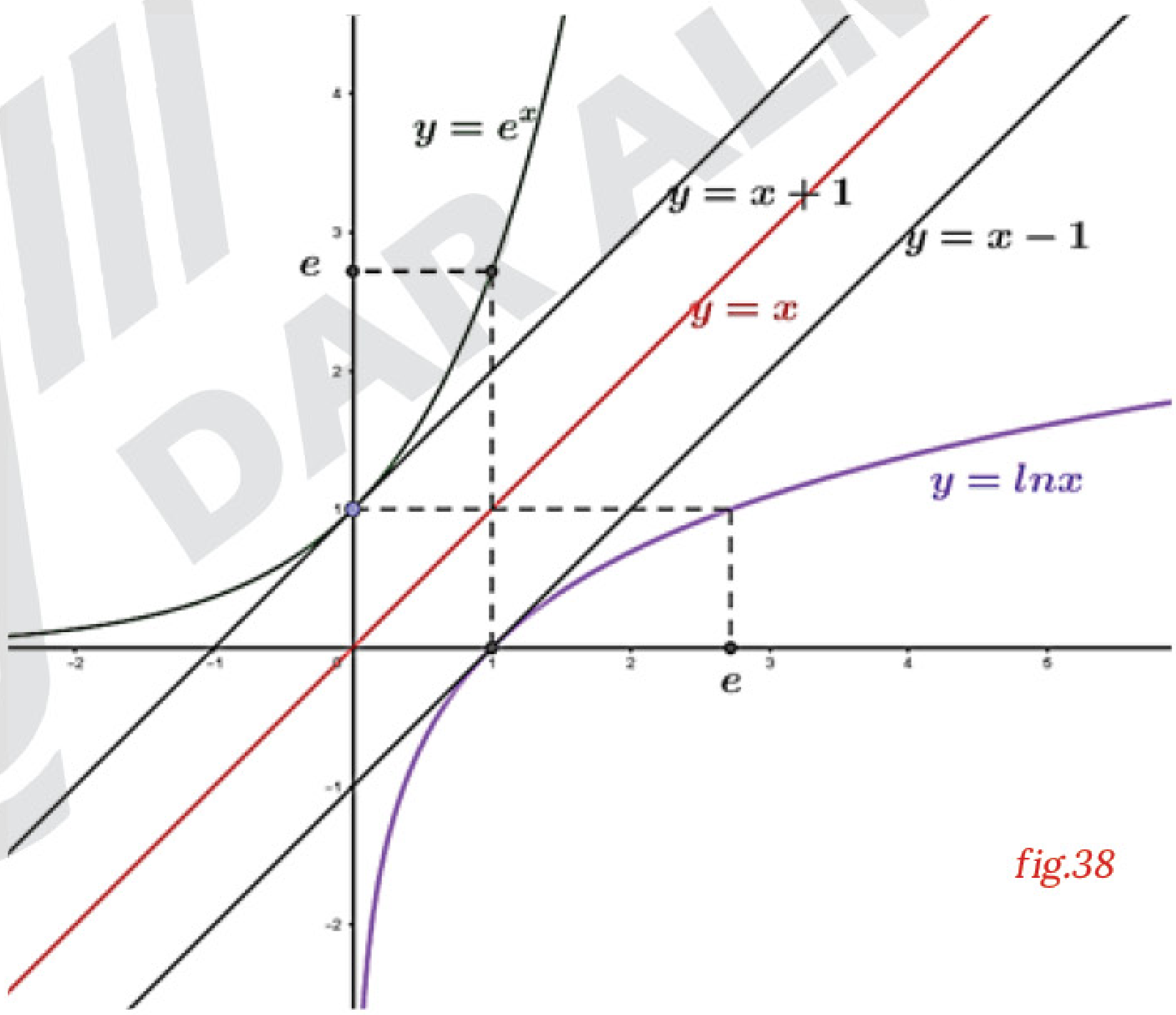

fig.38

b. Let us find now the general term of a geometric sequence:

$a_1 = q \cdot a_0$; $a_2 = q \cdot a_1 = q^2 \cdot a_0$; … ; prove that: $a_n = q^n \cdot a_0$

Assuming true for n and examine if it is true also for the consecutive order $n + 1$.

$a_n = q^n \cdot a_0$ then $a_{n+1} = q \cdot a_n = q \cdot q^n \cdot a_0 = q^{n+1} \cdot a_0$. True for all n.

4- Sum of consecutive terms:

a. Of an arithmetic sequence:

We know that: $a_n + a_0 = a_{n-1} + a_1 = a_{n-2} + a_2 = ...$ the sum of all two equidistant equal terms from a_0 and a_n.

From 0 to n we have $n + 1$ terms, thus: $\frac{n+1}{2}$ equal pairs.

Then the sum of all terms of the arithmetic sequence is:

$$S_n = \frac{n+1}{2}(a_0 + a_n) = \frac{n+1}{2}(a_0 + n.r.a_0).$$

b. Of geometric sequence:

$S_n = a_0 + a_1 + \dots + a_n.$

$qS_n = qa_0 + qa_1 + qa_2 + \dots + qa_n.$

The difference gives:

$(q-1)S_n = qa_n - a_0 = q^{n+1} \cdot a_0 - a_0$. And: $S_n = a_0 \frac{q^{n+1} - 1}{q - 1}$

$n, p \in \mathbb{N}, a, r, q \in \mathbb{R}$	Arithmetic sequence, reason r $n+1$ *terms* ; term $u_0 = a$	Geometric sequence, reason q and first term $u_0 = a$
Given by a recurrent relation	$u_0 = a$, $u_{n+1} = u_n + r$ $\forall n \in \mathbb{N}$	$u_0 = a$, $u_{n+1} = q.u_n$ $\forall n \in \mathbb{N}$
Given in explicit formula	$u_n = a + nr$	$u_n = a.q^n$
Relation between any two terms	$u_n = u_p + (n-p)r$	$u_n = u_p.q^{n-p}$
Sum of consecutive terms $S = u_p + u_{p+1} + \dots + u_n$	$S = \frac{(n-p+1)(u_p + u_n)}{2}$	$S = u_p \times \frac{1 - q^{n-p+1}}{1-q}$
Mean value	$u_n = \frac{u_{n+p} + u_{n-p}}{2}$	If $u_n > 0 \, \forall n \in \mathbb{N}$; $u_n = \sqrt{u_{n-p} \times u_{n+p}}$
Variations	If $r > 0$ then (u_n) is str ↗ If $r > 0$ then (u_n) is str ↘ If $r = 0$ then (u_n) is constant.	If $q > 1$ and $a > 0$ then (u_n) is ↗ If $q > 1$ and $a < 0$ then (u_n) is ↘ If $0 < q < 1$ and $a > 0$ *then* (u_n) is ↘ If $0 < q < 1$ and $a < 0$ *then* (u_n) is ↗ If $q = 1$ then (u_n) *is* constant. If $q < 0$ *then* u_n *is not* monotonic
Limit	If $r > 0$ then $\lim_{+\infty} u_n = +\infty$ If $r < 0$ then $\lim_{+\infty} u_n = -\infty$ If $r = 0$ then $lim u_n = a$	If $q > 1$ *then* $\begin{cases} lim u_n = +\infty & si\ a > 0 \\ lim u_n = -\infty & si\ a < 0 \end{cases}$ If $-1 < q < 1$ the $\lim_{+\infty} u_n = 0$ If $q \leq -1$, $(u_n)n$ has no limit

IV- Parallelism

Definition:

If (*d*) and (*d'*) are two parallel straight lines, any straight line (*d''*) parallel to one of them, is parallel to the second.
(*d*)//(*d'*) and (*d'*)//(*d''*) then (*d''*)//(*d*).

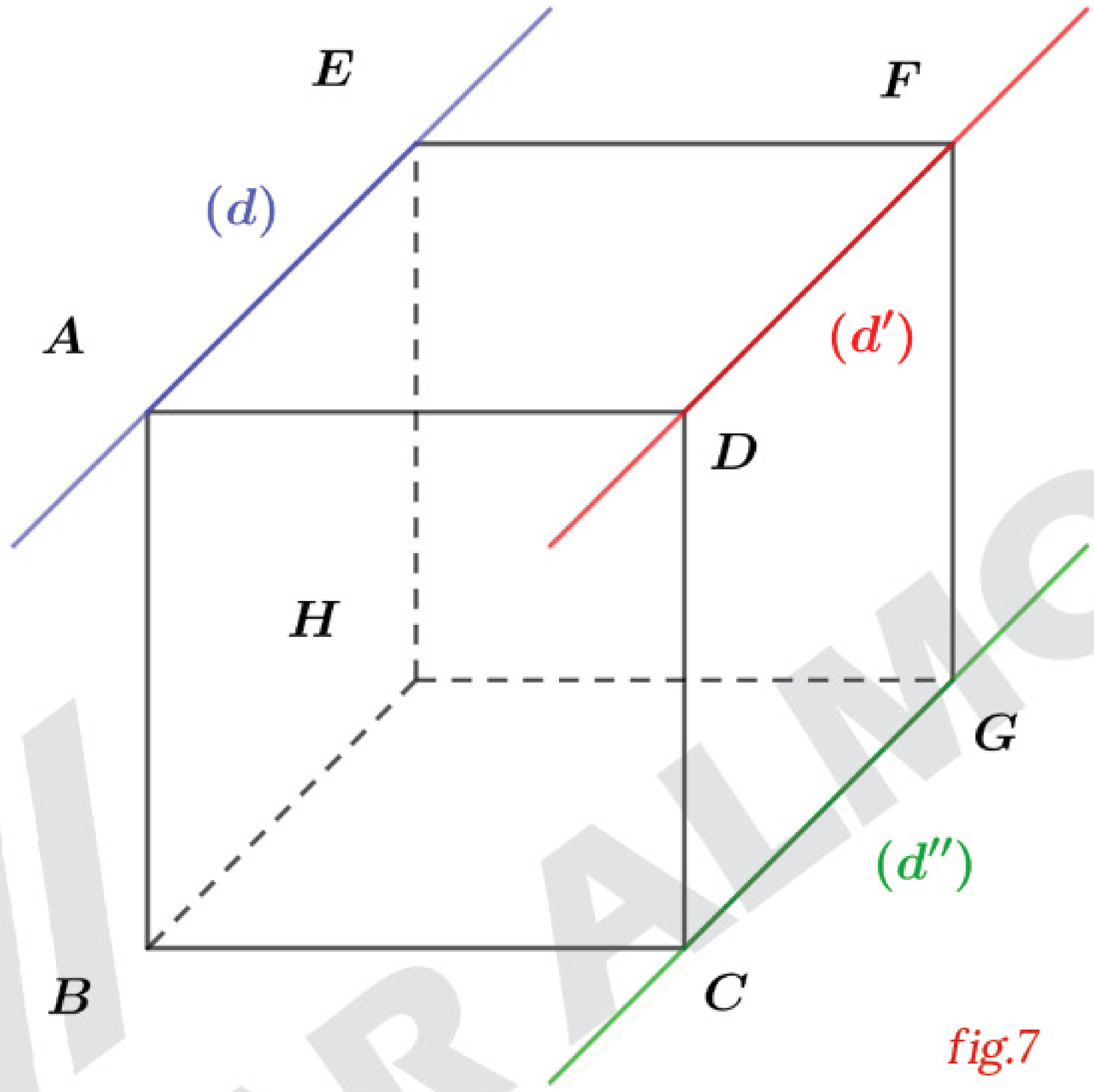

fig.7

If two planes (*P*) and (*Q*) are parallel, a plan (*R*) intersects (*P*) and (*Q*) in two parallel straight lines (*d*) and (*d'*).

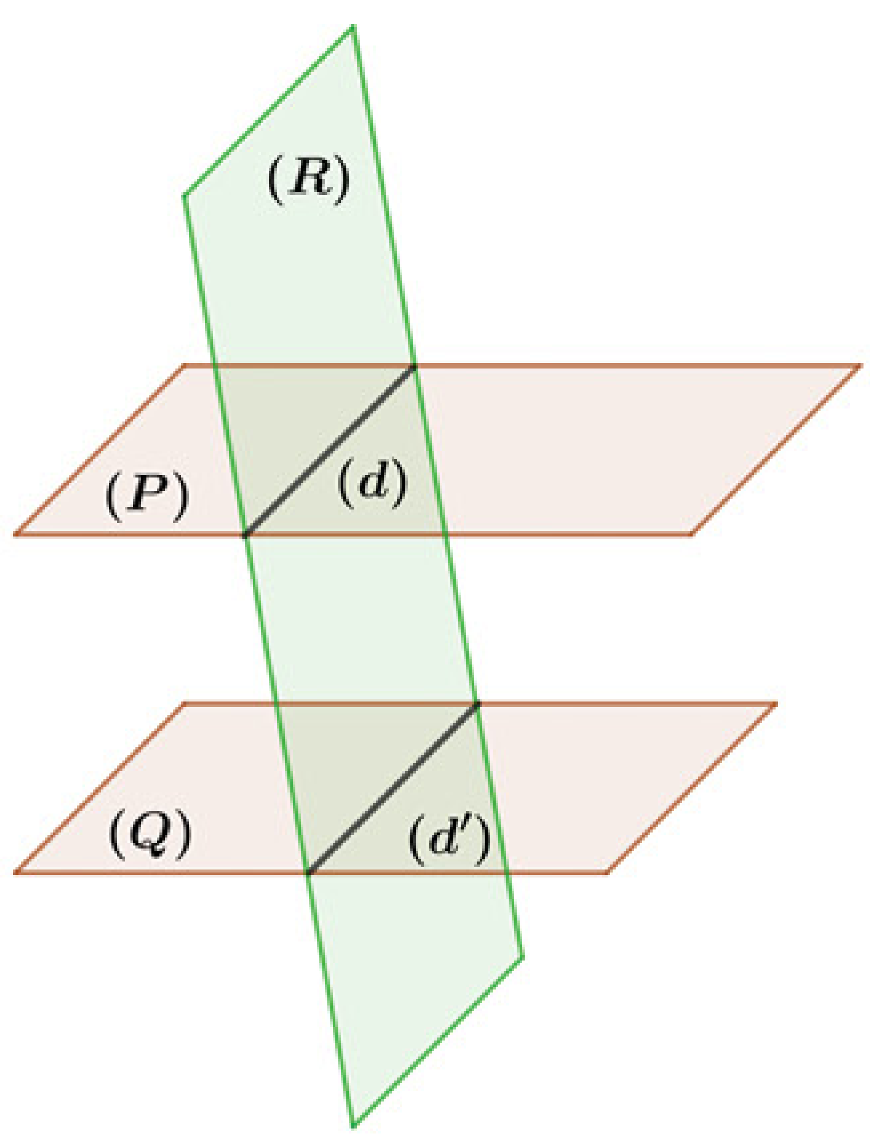

fig.8

➡ Example:

$ABCD$ is a square of center O and h is the dilation of center A which gives $O \to C$.
The image of (BD) by h is the parallel drawn from C to (BD) since:
$h(O) = C$ and $O \in (BD)$. Then
$h(D) = D'$ where $D' \in (AD)$ and $D' \in h(BD)$.
Since I', the image of I, is collinear with A and I (A center of h) then I' belongs to the parallel to (DC) drawn through D'.

The ratio of h is: $k = \frac{\overrightarrow{AC}}{\overrightarrow{AO}} = 2$.

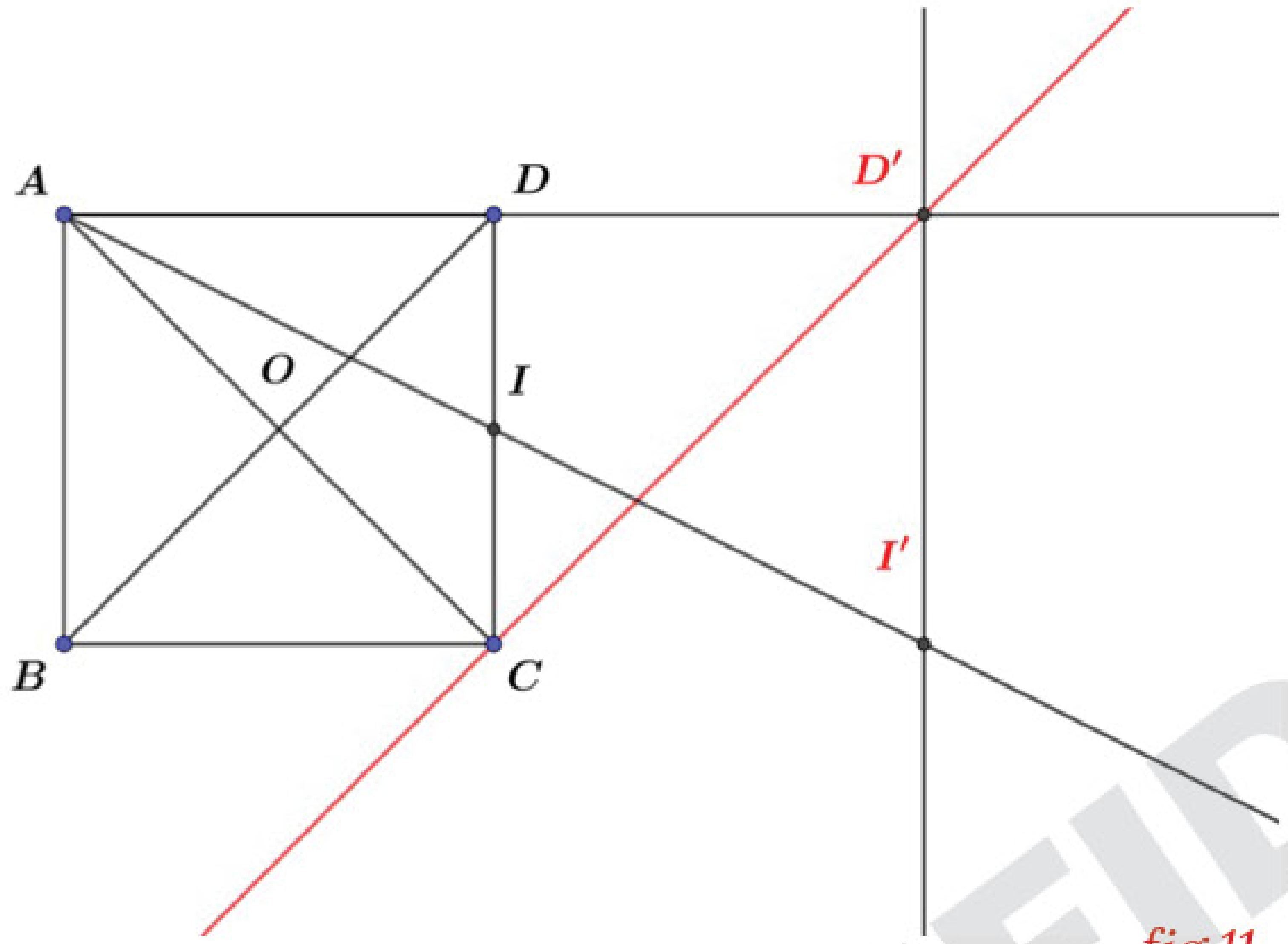

fig.11

VII- Direct similitude

1- Definition:

The transformation which associates to $M \to M'$ such that:

$$P \longrightarrow P$$
$$M \longrightarrow M'$$

for a given point Ω and an angle $\alpha\ (2\pi)$: $\begin{cases} \Omega M' = k\Omega M\ ;\ k > 0 \\ (\overrightarrow{\Omega M};\ \overrightarrow{\Omega M'}) = \alpha\ (2\pi) \end{cases}$

is called a similitude. Note it $S\ (\Omega;\ k;\ \alpha)$

Complex form: Let $M(z)$, $M'\ (z')$ and $\Omega(z_\Omega)$ be such that:

$z' - z_\Omega = ke^{i\alpha}\ (z - z_\Omega),\ k > 0$ by $S(\Omega;\ k;\ \alpha)$

$z' = az + b$ with $a = ke^{i\alpha}$, and $z_\Omega = \frac{b}{1 - a}$

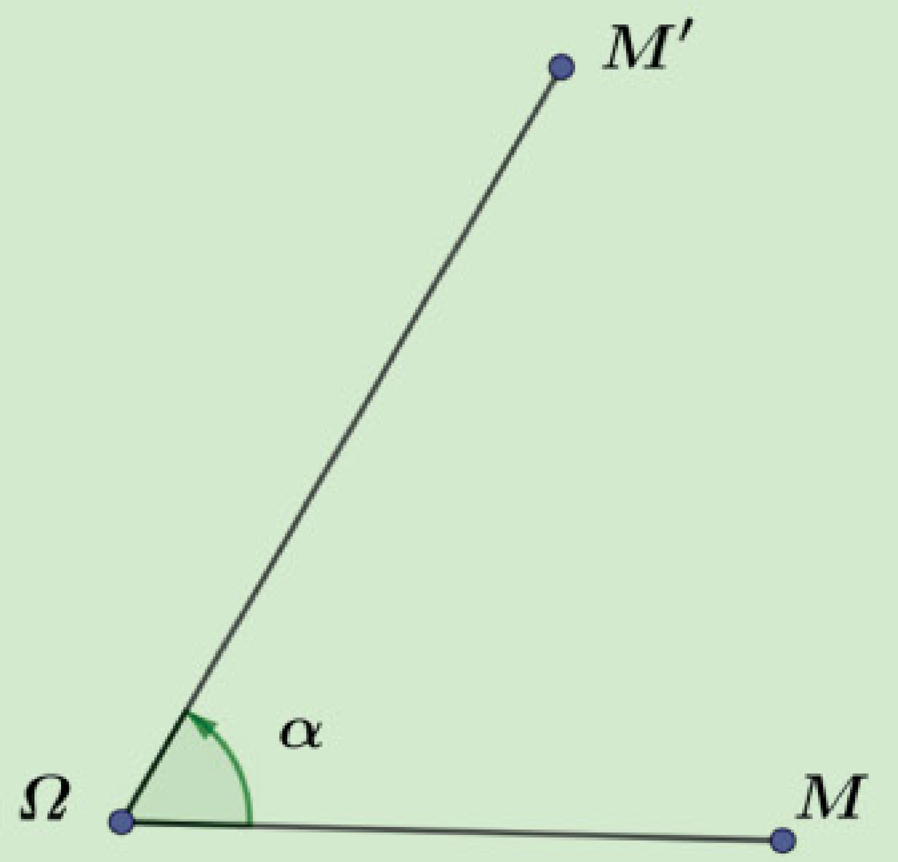

fig.12

Remark:
The analytic form can be deduced form the complex form.