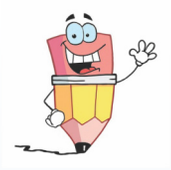
Educational intentions



Activities

Observe • Discover • Schedule the course

2 Finding the properties of a table of proportionality

Recopy and complete the tables of proportionality below by applying the proposed calculation methods :

Apples	× …	
Mass (in kg)	7	4
Price (in LL)	21 000	

Bananas	+		
Mass (in kg)	5	3	8
Price (in LL)	7 500	4 500	

Knowledge

Know the basics:
Control (Know): rules, formulas and theorems

II The properties of a triangle

* Metric property (inequality triangular)

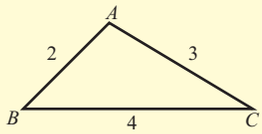
For any triangle, the length of each side is less than the sum of the lengths of the two remaining sides.

So, for example, in the triangle ABC, we have :

* $BC < BA + AC$: $4 < 2 + 3$

* $AB < AC + CB$: $2 < 3 + 4$

* $AC < AB + BC$: $3 < 2 + 4$



How to do

Know how to apply the course • Write a solution

3 Knowledge Solve an equation of the type $a\,x + b = c\,x + d$

Statement : Solve an equation $5\,x + 2 = 2\,x - 10$.

Solution :

Resolution by successive equivalent equations	Comments
$5\,x + 2 = 2\,x - 10$ $5\,x - 2\,x = -10 - 2$	We isolate the unknown terms in a side, and the known terms in the other side.
$3\,x = -12$	We reduce each side.
$\frac{3x}{3} = \frac{-12}{3}$	We divide the two sides by the coefficient of x (being non-zero)
$x = -4$	We give the solution.

Educational intentions

How to Apply

Apply • Practice • Find methods

7 1° Copy and complete the table according to the proposed model :

Proportions	First term	Second term	Third term	Fourth term
$\frac{6}{9}=\frac{8}{12}$	6	9	8	12
$\frac{6}{8}=\frac{9}{12}$	...	...	...	...
$\frac{12}{9}=\frac{8}{6}$	...	...	...	...
...	9	6	12	8

2° Name the mean terms and the extreme terms of the proportion $\frac{6}{9}=\frac{8}{12}$.

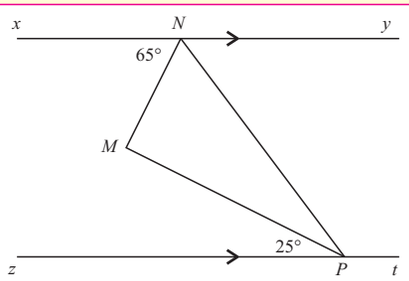
Know-how to be Competent

Develop • Look for • Find evaluate knowledge

26 In the adjacent figure, we have :

- $(xy) \,//\, (zt)$
- $\widehat{MNx} = 65°$
- $\widehat{MPz} = 25°$.

Prove that MNP is a right triangle at M.



Reasoning

True / False • MCQ • Justify • Prove

12 ATM is a right triangle at T such that ; $TA = 5$; $TM = 12$; $AM = 13$ (in cm).

We designate by P the perimeter of this triangle and by S its area.

For each question choose the good answer with justification :

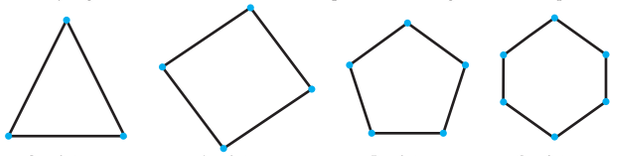
1° P = ... ? 60 cm 60 cm^2 30 cm 30 cm^2.

2° S = ... ? 60 cm 60 cm^2 30 cm 30 cm^2.

Just for fun

Number of segments

a. State how many segments can be drawn between the points in each figure. No three points are collinear.



3 points
? segments

4 points
? segments

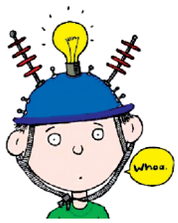
5 points
? segments

6 points
? segments

Chapter 4

The powers

The wheat on the chessboard



Knowledge

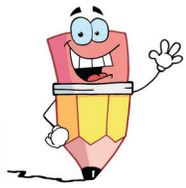
I. Square and Cube of a number.
II. Power of a number.
III. First rule of priority.
IV. Second rule of priority.
V. Rules of calculation.
VI. Powers of the number 10.

How to do

Calculate with applying the rules of priority.
Calculate with parentheses.
Decompose a power in a product of powers.
Calculate the powers in two ways.
Multiply a number by a power of 10.
Divide a number by a power of 10.

Chapter 4

The powers

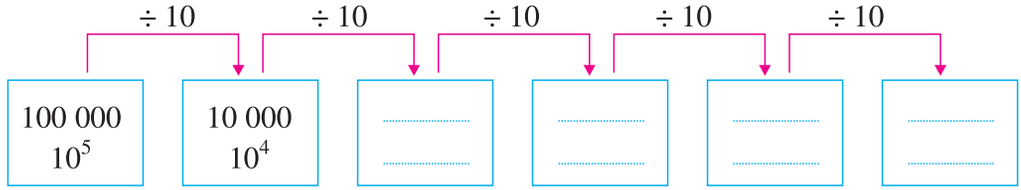


Activities

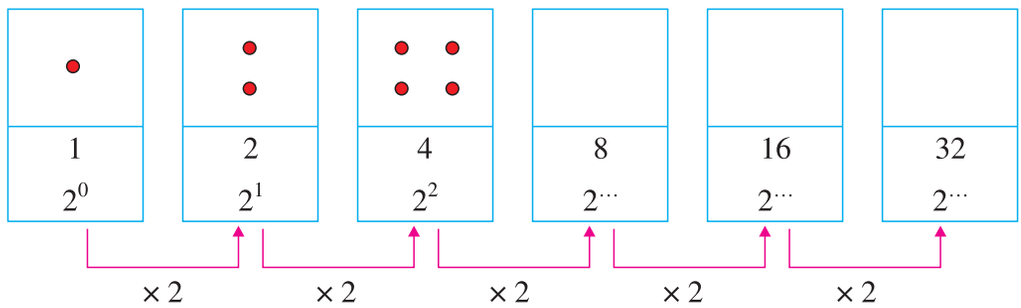
1 **Match each expression to its numerical value :**

The double of 5	•	•	10
The square of 5	•	•	15
The triple of 5	•	•	25
The cube of 5	•	•	125

2 **Observe, then complete as the same way :**



3 **Observe, then complete :**



4 **Observe the equalities, then complete as given :**

$10^3 \times 10^2 = 10^{3+2}$

* $10^3 \times 10^2 = (10 \times 10 \times 10) \times (10 \times 10) = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$.
* $2^3 \times 2^4 =$ .....
* $1.5^3 \times 1.5^2 =$ .....

5 **Observe the equalities, then complete as given :**

$(10^3)^2 = 10^{3\times 2}$

* $(10^3)^2 = 10^3 \times 10^3 = 10^{3+3} = 10^6$.
* $(2^4)^2 =$ .....
* $[1.5^2]^3 =$ .....

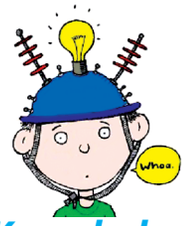
6 **Observe the equalities, then complete as given :**

$(2 \times 10)^3 = 2^3 \times 10^3$

* $(2 \times 10)^3 = (2 \times 10) \times (2 \times 10) \times (2 \times 10) = (2 \times 2 \times 2) \times (10 \times 10 \times 10) = 2^3 \times 10^3$.
* $(3 \times 5)^2 =$ .....
* $(4 \times 7)^3 =$ .....

Chapter 4

The powers



Knowledge

I Square and cube of a number

* The square of a number a, designated a^2, is the product of this number by itself :

$$a^2 = a \times a$$

$a^2 = a \times a$

Examples: $10^2 = 10 \times 10 = 100$; $(1.5)^2 = 1.5 \times 1.5 = 2.25$.

* The cube of a number a, designated a^3, is the product of three factors equal to this number :

$$a^3 = a \times a \times a$$

$a^3 = a \times a \times a$

Examples: $10^3 = 10 \times 10 \times 10 = 1\,000$; $(1.5)^3 = 1.5 \times 1.5 \times 1.5 = 3.375$.

II Power of a number

Let a be any number and n is a natural number greater than 1 ($n > 1$).

* The notation power a^n represents the product of n factors, all are equal to the number a :

$$a^n = \underbrace{a \times a \times \ldots \times a}_{n \text{ factors}}$$

a^n → exposent, base

Vocabulary: a^n is the power of base a and exponent n.

Example: The power of base 2 and exponent 5 is written 2^5.

We have : $2^5 = \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{5 \text{ factors}} = 32$.

The writing 0^0 has no sense

* Two particular cases : $0^n = 0$ and $1^n = 1$

* Two conventions : $a^1 = a$ and $a^0 = 1$ $(a \neq 0)$

Examples: $7^0 = 1$; $7^1 = 7$; $(1.5)^0 = 1$; $(1.5)^1 = 1.5$.

III First rule of priority

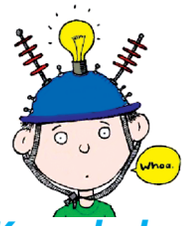
In the abscence of parentheses, the calculation of powers has the priority on the four elementary operation. (+ ; − ; × ; ÷).

Examples:

$A = 20 + 2^3$	$B = 6^2 - 3^3$	$C = 2.5 \times 10^2$	$D = 6^3 \div 2^4$
$A = 20 + 8$	$B = 36 - 27$	$C = 2.5 \times 100$	$D = 216 \div 16$
$A = 28$	$B = 9$	$C = 250$	$D = 13.5$

Chapter 4

The powers



Knowledge

IV Second rule of priority

In an expression containing parentheses, the calculation inside the parentheses has the priority over the rest.

Example: Calculate the expression : $F = (5^2 - 3 \times 2^3)^7$.

First step	Second step	Third step
$F = (25 - 3 \times 8)^7$	$F = (25 - 24)^7$	$F = (1)^7 = 1$

V Rules of calculation

We designate by a and b any two non-zero numbers, and by m and n two natural numbers.

	Formulas	Examples
Product of powers (same base)	$a^m \times a^n = a^{m+n}$	$7^2 \times 7^3 = 7^{2+3} = 7^5$ $2 \times 2^9 = 2^{1+9} = 2^{10}$
Power of a power	$(a^m)^n = a^{m \times n}$	$(7^2)^3 = 7^{2 \times 3} = 7^6$ $(2^3)^5 = 2^{3 \times 5} = 2^{15}$
Power of a product (same exponent)	$(a \times b)^m = a^m \times b^m$	$(2 \times 5)^3 = 2^3 \times 5^3$ $2^4 \times 7^4 = (2 \times 7)^4$

VI Powers of the number 10

We designate by n a natural greater than 1.

* $10^n = \underbrace{10 \times 10 \times \ldots\ldots \times 10}_{n \text{ factors}} = \underbrace{100\ldots\ldots0}_{n \text{ zeros}}$.

n is a natural number :
10^n, is 1 followed by n zeros

Example: $10^4 = \underbrace{10 \times 10 \times 10 \times 10}_{4 \text{ factors}} = \underbrace{10\,000}_{4 \text{ zeros}}$

* Two rules of calculation : $10^m \times 10^n = 10^{m+n}$ $\quad$ $(10^m)^n = 10^{m \times n}$

Examples: $10^2 \times 10^4 = 10^{2+4} = 10^6$; $(10^2)^4 = 10^{2 \times 4} = 10^8$.

* Two conventions : $10^1 = 10$ and $10^0 = 1$.

10^2 : a hundred
10^3 : a thousand
10^6 : a million
10^9 : a billion

Chapter 4

The powers

How to do

1 Knowledge Calculate with applying the rules of priority.

Statement : Calculate the numerical expression : $E = 6 + 6^3 \div 4 - 3 \times 2^4$.

Solution :

Method	Execution
With the abscence of parentheses, we perform in the order : * The power → * The multiplications and the divisions → * The additions and the subtractions →	$E = 6 + 6^3 \div 4 - 3 \times 2^4$ $E = 6 + 216 \div 4 - 3 \times 16$ $E = 6 + 54 - 48$ $E = 60 - 48$ $E = 12$

2 Knowledge Calculate with parentheses.

Statement :
1. Compare the numbers : $a = (5 + 3)^2$ and $b = 5^2 + 3^2$.
2. Compare the numbers : $c = (7 - 4)^3$ and $d = 7^3 - 4^3$.

Solution :

1.	$a = (5 + 3)^2$ $a = 8^2$ $a = 64$	$b = 5^2 + 3^2$ $b = 25 + 9$ $b = 34$	Since $64 > 34$ then $a > b$
2.	$c = (7 - 4)^3$ $c = 3^3$ $c = 27$	$d = 7^3 - 4^3$ $d = 343 - 64$ $d = 279$	Since $27 < 279$ then $c < d$

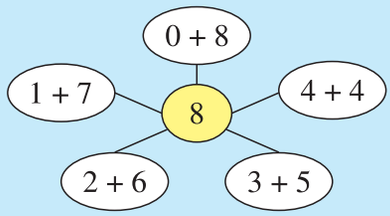
Remark: In a general way : $(a + b)^n \neq a^n + b^n$ and $(a - b)^n \neq a^n - b^n$

3 Knowledge Decompose a power in a product of powers.

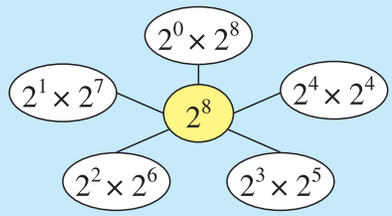
Statement : Write the power 2^8 in the form of a product of two powers of the number 2.

Solution

* Write the exposent 8 in the form of a sum at two natural numbers :



* We deduce :



Method : If n, p and q are three natural numbers such as $n = p + q$, then $a^n = a^p \times a^q$ for every non-zero number a.

Chapter 4

The powers

How to do

4 Knowledge Calculate in two ways the powers.

Statement : Calculate in two ways the expression : $E = 4^6 \times 0.5^6$.

Solution :

First way

Apply the priority rule :

$E = 4^6 \times 0.5^6$
$E = 4\,096 \times 0.015\,625$
$E = 64$

Second way

We apply the rules of priority :

$E = 4^6 \times 0.5^6$
$E = (4 \times 0.5)^6$
$E = 2^6$
$E = 64$

It is the easiest

5 Knowledge Multiply a number by a power of 10.

Method : Multiply a decimal by 10^n, with n a natural number, is to move the comma to the right, with adding zeros if necessary.

Statement : Calculate the products : $P = 25 \times 10^4$; $R = 3.25 \times 10^5$

Solution :

* Product	Technique	Result
$P = 25 \times 10^4$	25 000 000 (4 ranks)	$P = 250\,000$
* Product	Technique	Result
$R = 3.25 \times 10^5$	3.250 000 (5 ranks)	$R = 325\,000$

6 Knowledge Divide a number by a power of 10.

Method : Divide a decimal by 10^n, with n a natural number, is to move back the comma to the left, with adding zeros if necessary.

Statement : Calculate the quotients : $Q = 3\,750 \div 10^6$; $S = 8.25 \div 10^3$.

Solution :

* Product	Technique	Result
$Q = 3\,750 \div 10^6$	00 003 750 (6 ranks)	$Q = 0.003\,75$
* Product	Technique	Result
$S = 8.25 \div 10^3$	00 008.25 (3 ranks)	$S = 0.008\,25$

Chapter 4

The powers

How to Apply

Calculations and priorities

1 Copy and complete the following sentences :

a) 5^4 is the power of base .... and exponent ....

b) 4^5 is the product of .... factors equal to the number ....

2 Write each number using power notation :

$a = 4 \times 4 \times 4$; $b = 3 \times 3 \times 3 \times 3$; $c = 3 + 3 + 3$; $d = 4 + 4 + 4 + 4$.

3 1° Calculate the square of 4, then the cube of 5.

2° Deduce without calculation : 40^2 ; 0.4^2 ; 50^3 ; 0.5^3 .

4 1° Calculate each of the following expressions :

$A = 7^1 - 1^7 + 7^0$; $B = 2\,020^1 + 2\,020^0 - 1^{2\,020}$.

2° Can we calculate $C = (7 - 2 \times 3.5)^0$? Why ?

5 1° Calculate : 7^2 ; 7^3 ; 1.5^2 ; 1.5^3 .

2° Use the previous results to verify the equality :

$7^3 - 1.5^3 = (7 - 1.5) \times (7^2 + 7 \times 1.5 + 1.5^2)$.

6 Verify / check that the following expressions are four different writings form of the number 365 :

$E = 10^2 + 11^2 + 12^2$; $F = 13^2 + 14^2$

$G = (1^2 + 2^2) \times (3^2 + 8^2)$; $H = (13 + 14)^2 - 2 \times 13 \times 14$.

7 1° Calculate : 4^0 ; 4^1 ; 4^2 ; 4^3 ; 4^4 .

2° Verify the equality : $(4^3 + 4^2 + 4^1 + 4^0) \times (4^1 - 4^0) = 4^4 - 1$.

Chapter 4

The powers

How to Apply

Apply the rules of calculation

8 Write each expression as a single power :

1° $A = 7^2 \times 7^4$; $B = 7^3 \times 7^3$; $C = 7 \times 7^3 \times 7^2$; $D = 7^4 \times 7^4 \times 7^4$.

2° $E = (7^2)^4$; $F = (7^3)^3$; $G = (7 \times 7^3)^2$; $H = (7^4 \times 7^4)^4$.

9 1° Calculate : 5^2 ; 5^3 ; 5^4.

2° Write each product in the form of a power of 5 :

$M = 25 \times 125$; $N = 25 \times 125 \times 625$; $P = 25^3 \times 125^2$.

10 1° Calculate : 3^0 ; 3^1 ; 3^2 ; 3^3 ; 3^4.

2° Use the previous results to verify the following equalities : a) $9^6 = 27^4$; b) $(3^3)^4 = 81^3$.

11 Write without parentheses each of the following expressions :

$U = (2 \times x)^2$; $V = (2 \times x)^3$; $W = (2 \times x)^4$

$X = (3 \times a \times b)^2$; $Y = (3 \times a \times b)^3$; $Z = (3 \times a \times b)^4$.

12 Express in the form of a^n where a and n are natural numbers :

$I = 2^4 \times 7^4$; $J = 2^5 \times 3^5 \times 5^5$; $K = 4^6 \times 1.5^6$; $L = 2^3 \times 3^3 \times 2.5^3$.

13 Cleverly calculate each of the following products :

1° $R = 2^7 \times 0.5^7$; $S = 0.4^{12} \times 2.5^{12}$ 2° $T = 4^6 \times 0.5^6$; $U = 0.8^5 \times 2.5^5$.

14 Write the power of 3^5 in the form $3^p \times 3^q$ where p and q are two natural numbers.
Give all the possibilities.

15 1° Verify that : $0.2^6 \times 5^6 = 1$.

2° Use the previous result to calculate the following expressions :

$P = 0.2^6 \times 5^7$; $Q = 0.2^7 \times 5^6$; $R = 0.2^6 \times 5^8$; $S = 0.2^8 \times 5^6$.

16 Write each of the following doubles as a power of 2 :

$D = 2^5 + 2^5$; $E = 2^{12} + 2^{12}$; $F = 2^{25} + 2^{25}$.

Chapter 4

The powers

How to Apply

Powers of 10

17 Write each of the following numbers as a power of 10 :

1° ten thousands ; hundred thousands ; ten millions ; hundred millions.

2° 1 000 ; 1 000 000 ; 100 000 000 ; 10 000 000 000 .

18 Show that each of the following expressions represents a power of 10 :

1° $A = 10^3 \times 10^5$; $B = 10^7 \times 10^7$; $C = 10 \times 10^3 \times 10^2$.

2° $D = (10^3)^5$; $E = (10^7)^2$; $F = (1 \times 10^3)^2$.

19 Give the decimal form of each of the natural numbers :

$M = 3 \times 10^5 + 7 \times 10^3 + 4 \times 10^1$; $N = 8 \times 10^8 + 6 \times 10^6 + 4 \times 10^4 + 2 \times 10^2$.

20 Give the decimal writing of each of the numbers :

1° $m = 0.275 \times 10^2$; $n = 0.275 \times 10^4$. 2° $p = 3\,500 \div 10^3$; $q = 3\,500 \div 10^5$.

Know-how to be Competent

21 Answer by true or false :

1° The expression $(1 + 2)^2$ is equal to $1^2 + 2^2$.

2° The expression $(1 + 2)^2$ is equal to $1^3 + 2^3$.

3° The cube of 4 is equal to the square of 8.

4° $3\,750\,000 \div 10^3 = 3.75 \times 10^3$.

22 In each case, copy and complete the equality with the correct integer :

a) $12 \times 12^3 \times 12^{\cdots} = 12^8$ b) $3^5 \times 7^{\cdots} = 21^5$

c) $(7 - \ldots)^5 = 0$ d) $(7 - \ldots)^5 = 1$ e) $(7 - 5)^{\cdots} = 1$

23 For each question only one answer is correct.
Choose the right answer by justifying the choice :

1°	$15^2 - 10^2 = \ldots$	a) 5	b) 5^2	c) 5^3
2°	$10^{12} = \ldots$	a) $10^6 + 10^6$	b) $(10^6)^2$	c) 2×10^6
3°	$7^3 = \ldots$	a) $7 + 7 + 7$	b) 7×7^2	c) 7×3
4°	2 500 can be ...	a) $2.5 \div 10^3$	b) 25×10^3	c) 2.5×10^3

Chapter 4

The powers

Know-how to be Competent

24 Calculate each of the following numerical expressions :

$A = (4^3 + 2.5^3) \div (4^2 + 2.5^2 - 4 \times 2.5)$

$B = (2^3 - 2 \times 3)^5 \div (2^3 - 2 \times 3)^3$

$C = (2 \times 4 \times 5)^2 - 2 \times (4 \times 5)^2 - 2 \times 4 \times 5^2$

$D = 8^3 + 3 \times 8 \times 7^2 - 3 \times 8^2 \times 7 - 7^3$.

25 Translate each sentence into a numerical expression, then calculate :

A : The double of the square of 6 B : The square of the double of 6

C : The triple of the cube of 6 D : The cube of the triple of 6.

26 Consider the two numbers : $a = 2.5 \times 10^5$ and $b = 4 \times 10^4$.

Verify that : i. $a + b = 29 \times 10^4$; ii. $a - b = 21 \times 10^4$

iii. $a \times b = 10^{10}$; iv. $a \div b = 625 \div 10^2$.

27 Recopy and complete each equality by the suitable exponent :

a) $0.000\,375 \times 10^{\cdots} = 375$; b) $6\,500 \div 10^{\cdots} = 6.5$

c) $8.25 \times 10^{\cdots} = 82\,500$; d) $23\,500 \div 10^{\cdots} = 0.0235$.

28 Calculate the difference of half the square of 12 and the square of the half of 12.