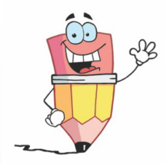
Educational intentions

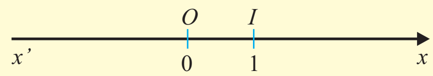


Activities

Observe • Discover • Schedule the course

5 Consider the axis $x'Ox$

Copy and compete by $\leq$ or $\geq$:



a) The semi-straight line $[Ox)$ represents the numbers x such that $x \ldots 0$.

b) The semi-straight line $[Ox')$ represents the numbers x such that $x \ldots 0$.

Knowledge

Know the basics:
Control (Know): rules, formulas and theorems

III Length of an arc of circle

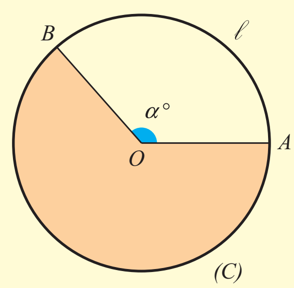
In a circle, the length of an arc is propotional to the central angle intercepting its arc.

The circle $C(O\,;R)$, of length $2\pi R$, is an arc measuring 360°.

Denoted by ℓ the length of arc $\overset{\frown}{AB}$, and given $\widehat{AOB} = \alpha°$, we have the proportionality table below :

Measure of an arc	360°	$\alpha°$
Length of an arc	$2\pi R$	ℓ

We can conclude the calculation rule : $\ell = 2\pi R \times \dfrac{\alpha°}{360°}$.



How to do

Know how to apply the course • Write a solution

2 Knowledge Solve the equation of type: $x^2 = a$

Statement Solve the equations : $x^2 = 3$; $(x-1)^2 = 5$.

Solution

- Since $3 > 0$, then $x^2 = 3$ gives $x = -\sqrt{3}$ or $x = +\sqrt{3}$.
- Since $5 > 0$, then $(x-1)^2 = 5$ gives $x - 1 = -\sqrt{3}$ or $x - 1 = \sqrt{5}$.

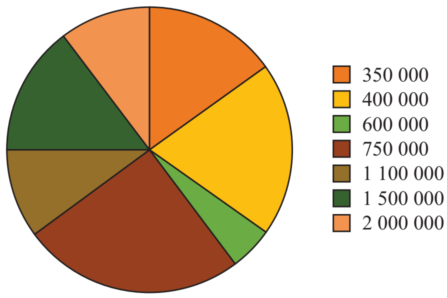
  We can conclude that : $x = 1 - \sqrt{5}$ or $x = 1 + \sqrt{5}$.

Educational intentions

How to Apply

Apply • Practice • Find methods

4 A survey of salaries in L.L. of 540 people showed the following results given as a circular diagram.



1° Draw up a table showing the frequencies and the cumulative frequencies.
2° Draw the bar chart and the polygon of the cumulative frequencies.

Know-how to be Competent

Develop • Look for • Find evaluate knowledge

14 Draw a triangle *ABC*.
Let *T* be the translation of vector $\overrightarrow{BC}$.
We denoted by *E* the image of *A* by the translation *T*, *F* is the image *E* by the translation *T* and *D* the image of *F* by the translation *T*.
1° Construct the three points *E*, *F* and *D* and show that they are collinear.
2° Show that *ABCD* is a trapezoid and calculate the average of the bases in terms of *BC*.
3° Express the area of this trapezoid according to that of triangle *ABC*.

Reasoning

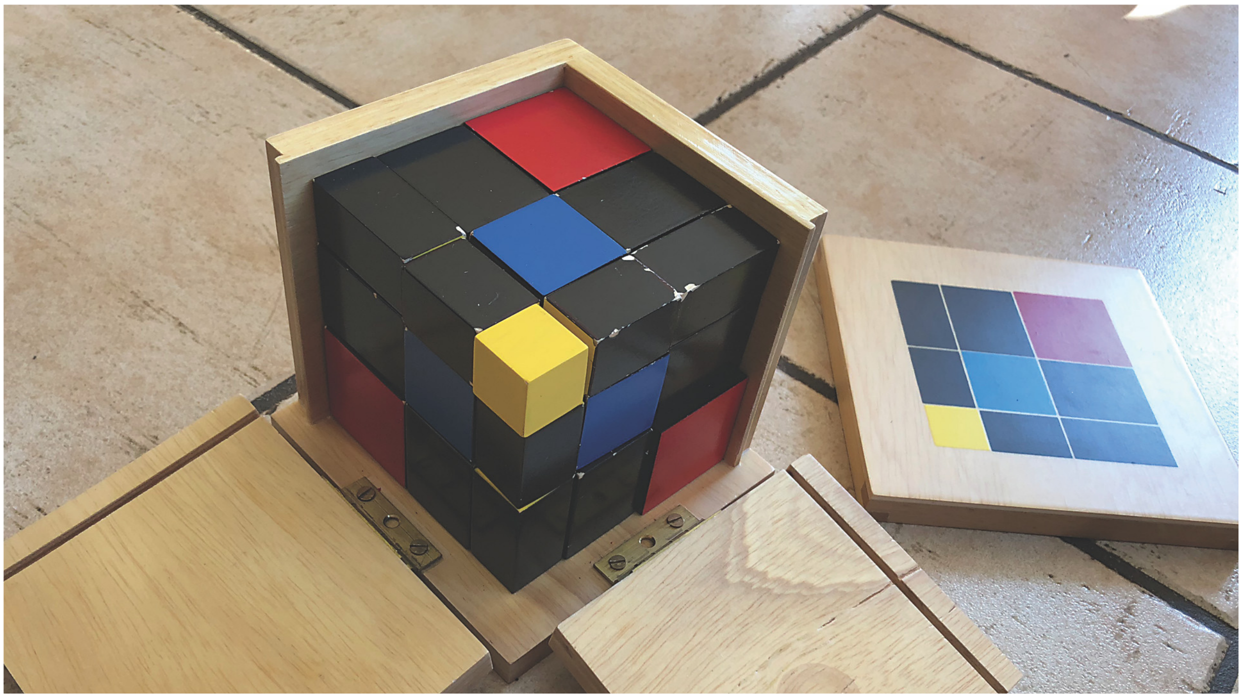
True / False • MCQ • Justify • Prove

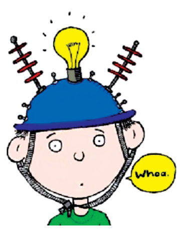
24 ***MCQ*** For each question, choose the right answer and justify.

	Answer *A*	Answer *B*	Answer *C*
1° $\sqrt{15^2 + 8^2}$ = ...	15 + 2	15 + 4	15 + 8
2° $\left(\sqrt{2}\right)^2 \times \left(\sqrt{2}\right)^3$ = ...	$\left(\sqrt{2}\right)^6$	$4\sqrt{2}$	$8\sqrt{2}$
3° For $a > 0$, $\frac{a}{\sqrt{a}}$ = …	1	$\frac{\sqrt{a}}{2}$	$\sqrt{a}$
4° If $a = 10^{16}$, then $\sqrt{a}$ = …	10^8	10^{14}	10^4

Chapter 7

Remarkable identities





Knowledge

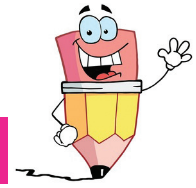
I. Vocabulary
II. First remarkable identity : Square of a sum
III. Second remarkable identity : Square of a difference
IV. Third remarkable identity : Product of a sum by a difference.

How to do

Establishing and using two particular identities
To factorize an expression of the form $a^2 + b^2 + 2ab$
To factorize an expression of the form $a^2 + b^2 - 2ab$
To factorize an expression of the form $a^2 - b^2$ (difference of two squares)
Factorization by steps.

Chapter 7

Remarkable identities



Activities

1 **Consider the two numbers : $a = 5$ and $b = 3$.**

1° Calculate : $(a+b)^2$; $(a-b)^2$; a^2+b^2 ; a^2-b^2.

2° Copy and complete using = or ≠ :

i. $(a+b)^2$ …… a^2+b^2 ; ii. $(a-b)^2$ …… a^2-b^2

iii. $(a-b)^2$ …… a^2+b^2 ; iv. a^2+b^2 …… a^2-b^2

3° i. Calculate : ab ; $2\,ab$; $2a \times 2b$.

ii. Complete using = or ≠ : $2ab$ …… $2a \times 2b$.

2 **Consider the algebraic expressions : $A = (x-1)(x^2+x+1)$ and $B = x^3 - 1$.**

1° For $x = 2$, verify that : $A = B$.

2° True or false ?

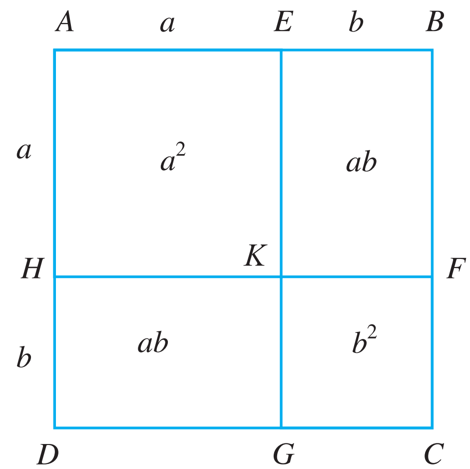
The algebraic expressions A and B are identical for any value of x.

3 **The square $ABCD$ has for side $(a + b)$.**

1° Write its area S using a and b.

2° The area S is also equal to the sum of the areas of two squares $AEKH$ and $GCFK$, and of the two rectangles $DGKH$ and $EBFK$.

Verify that : $(a+b)^2 = a^2 + b^2 + 2ab$.



4 **Consider any two numbers a and b.**

1° Perform and reduce the product $P = (a+b)(a-b)$.

2° Calculate the product $Q = (5+3)(5-3)$ in two different ways.

5 **Consider any two numbers a and b.**

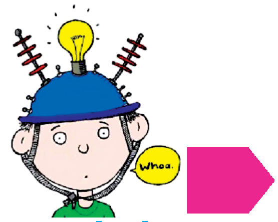
Copy and complete using = or ≠ :

i. $(-a)(-b)$ …… $-ab$; ii. $(-a)(-b)$ … ab

iii. $(-a-b)(a-b)$ …… $-(a+b)(a-b)$; iv. $(a-b)(b-a)$ …… $-(a-b)^2$.

Chapter 7

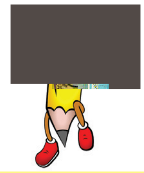
Remarkable identities



Knowledge

I Vocabulary

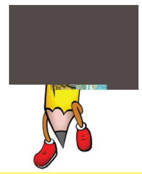
Designate by a and b two signed numbers.



$(a+b)^2 \neq a^2+b^2$

❶ Square of a sum ; sum of squares

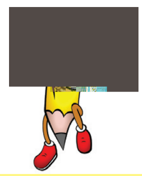
- The square of the sum of a and b is denoted $(a+b)^2$.
- The sum of the squares of a and b is denoted a^2+b^2.



$(a-b)^2 \neq a^2-b^2$

❷ Square of a difference ; difference of squares

- The square of the difference of a and b is denoted $(a-b)^2$.
- The difference of the squares of a and b is denoted a^2-b^2.



$2ab = 2 \times a \times b$

❸ Double – product of two numbers

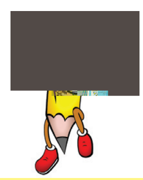
The expression $2\,ab$ represents the double-product of the two numbers a and b.

❹ Identity

The literal equality $a^2b - ab^2 = ab(a-b)$ is an identity :

It is verified for any numerical value given to the letters a and b.

II First remarkable identity : Square of a sum



$(a+b)^2 = a^2+b^2+2ab$

* The square of the sum of two numbers is equal to the sum of the squares of these two numbers increased by their double-product.

Justification : $(a+b)^2 = (a+b)\,(a+b) = a^2+ab+ab+b^2$
We deduce that: $(a+b)^2 = a^2+2ab+b^2$.

* To develop $(a+b)^2$, is to write it in the form a^2+b^2+2ab :

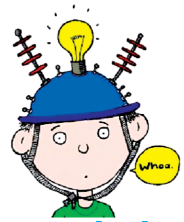
Square to be developed	Development technique	Developed form
$(2x+3)^2$	$(2x)^2+(3)^2+2(2x)\,(3)$	$4x^2+9+12x$

* Numerical application

To calculate mentally 52^2 for example, we notice that $52 = 50 + 2$.
So : $52^2 = (50+2)^2 = 50^2+2^2+2\,(50)\,(2)$.
We deduce that : $52^2 = 2\,500 + 4 + 200 = 2\,704$.

Chapter 7

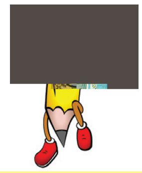
Remarkable identities



Knowledge

III Second remarkable identity : Square of a difference

* The square of a difference of two numbers is equal to the sum of the squares of these two numbers <u>decreased</u> by their double-product.



$(a - b)^2 = a^2 + b^2 - 2ab$

<u>Justification</u> : $(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2$
We deduce that: $(a - b)^2 = a^2 - 2ab + b^2$.

* <u>To develop</u> $(a - b)^2$, is to write in the form $a^2 + b^2 - 2ab$:

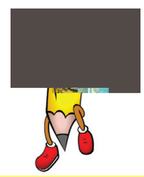
Square to be developed	Development technique	Developed form
$(2x - 3)^2$	$(2x)^2 + (3)^2 - 2(2x)(3)$	$4x^2 + 9 - 12x$

* <u>Numerical application</u>

To calculate mentally 48^2 for example, we notice that $48 = 50 - 2$.
So : $48^2 = (50 - 2)^2 = 50^2 + 2^2 - 2(50)(2)$.
We deduce that : $48^2 = 2\,500 + 4 - 200 = 2\,304$.

IV Third remarkable identity : Product of a sum by a difference

* For two numbers a and b, the product of the sum $(a + b)$ by the difference $(a - b)$ is equal to the difference $a^2 - b^2$ of their respective squares.



$(a + b)(a - b) = a^2 - b^2$

<u>Justification</u> : $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$.

* <u>To develop</u> the product $(a + b)(a - b)$, is to write it in the form $a^2 - b^2$:

Product to be developed	Development technique	Developed form
$(2x + 3)(2x - 3)$	$(2x)^2 - (3)^2$	$4x^2 - 9$

* <u>Numerical application</u>

To calculate mentally 52×48 for example, we notice that $52 = 50 + 2$ and $48 = 50 - 2$.
So : $52 \times 48 = (50 + 2)(50 - 2) = (50)^2 - (2)^2$.
We deduce that: $52 \times 48 = 2\,500 - 4 = 2\,496$.

Chapter 7

Remarkable identities

How to do

1 Knowledge — Establishing and using two particular identities

Statement

1° Prove the equalities : i. $(-a-b)^2 = (a+b)^2$; ii. $(-a+b)^2 = (a-b)^2$.

2° Develop and reduce the expression $P = (-2x-5)^2 - (-3x+4)^2$.

Solution

1° We know that two opposite numbers have the same square : $(-x)^2 = x^2$.

So :
- Since $(-a-b)$ is the opposite of $(a+b)$, then $(-a-b)^2 = (a+b)^2$.
- Since $(-a+b)$ is the opposite of $(a-b)$, then $(-a+b)^2 = (a-b)^2$.

2° We have : $P = (-2x-5)^2 - (-3x+4)^2 = (2x+5)^2 - (3x-4)^2$.

So : $P = \left[(2x)^2 + (5)^2 + 2(2x)(5)\right] - \left[(3x)^2 + (4)^2 - 2(3x)(4)\right]$

$P = \left[4x^2 + 25 + 20x\right] - \left[9x^2 + 16 - 24x\right]$

$P = 4x^2 + 25 + 20x - 9x^2 - 16 + 24x$

$P = -5x^2 + 44x + 9$.

2 Knowledge — To factorize an expression of the form $a^2 + b^2 + 2ab$

Method

To factorize an expression of the form $a^2 + b^2 + 2ab$.

Statement

1° Factorize the expression $E = 9x^2 + 25 + 30x$.

2° Deduce a writing of the expression $F = 18x^2 + 50 + 60x$ in the form $k(a+b)^2$.

Solution

1° We have : $9x^2 = (3x)^2$; $25 = (5)^2$; $30x = 2(3x)(5)$.

So : $E = (3x)^2 + (5)^2 + 2(3x)(5) = (3x+5)^2$.

2° $F = 18x^2 + 50 + 60x = 2(9x^2 + 25 + 30x)$.

So : $F = 2(3x+5)^2$, written in the form $k(a+b)^2$.

3 Knowledge — To factorize an expression of the form $a^2 + b^2 - 2ab$

Method

To factorize an expression of the form $a^2 + b^2 - 2ab$.

Statement

1° Factorize the expression $G = 4x^2 - 28x + 49$.

2° Deduce a writing of the expression $H = -4x^3 + 28x^2 - 49x$ in the form $k(a-b)^2$.

Solution

1° We have : $4x^2 = (2x)^2$; $49 = (7)^2$; $28x = 2(2x)(7)$.

So : $G = (2x)^2 + (7)^2 - 2(2x)(7) = (2x-7)^2$.

2° $H = -4x^3 + 28x^2 - 49x = -x(4x^2 - 28x + 49)$.

So : $H = -x(2x-7)^2$, written in the form $k(a-b)^2$.

Chapter 7

Remarkable identities

How to do

4 Knowledge To factorize an expression of the form $a^2 - b^2$ (Difference of two squares).

Method To factorize $a^2 - b^2$, is to write it in the form $(a + b)(a - b)$.

Statement Factorize the expression : $M = 4x^2 - 81$; $P = (-2x - 5)^2 - (-3x + 4)^2$.

Solution

- $M = 4x^2 - 81$
  $M = (2x)^2 - (9)^2$
  $M = (2x + 9)(2x - 9)$

- $P = (-2x - 5)^2 - (-3x + 4)^2$
  $P = [(-2x - 5) + (-3x + 4][(-2x - 5) - (-3x + 4)]$
  $P = [-2x - 5 - 3x + 4][-2x - 5 + 3x - 4]$
  $P = (-5x - 1)(x - 9)$

5 Knowledge Factorization by steps

Statement

1° Decompose the algebraic expression $A = (x - 3)^3 - 4(x - 3)$ into a product of three factors.

2° Factorize the expression $B = (6x - 10)^2 + (5 - 3x)(2x + 5) - 9x^2 + 25$.

Solution

1°

$A = (x - 3)^3 - 4(x - 3)$	**Commentaries**
$A = (x - 3)\left[(x - 3)^2 - 4\right]$	We take $(x - 3)$ as a common factor
$A = (x - 3[x - 3 + 2][x - 3 - 2]$	$(x - 3)^2 - 4$, is like $a^2 - b^2$
$A = (x - 3)(x - 1)(x - 5)$	We reduce the factors

2° First step

- We have : $6x - 10 = 2(3x - 5)$.
  So : $(6x - 10)^2 = [2(3x - 5)]^2 = 4(3x - 5)^2$.
- $5 - 3x$ is the opposite of $3x - 5$. So : $(5 - 3x) = -(3x - 5)$.
- $-9x^2 + 25$ is the opposite of $9x^2 - 25$.
  So : $-9x^2 + 25 = -(9x^2 - 25) = -(3x + 5)(3x - 5)$.

Second steps

$B = (6x - 10)^2 + (5 - 3x)(2x + 5) - 9x^2 + 25$

$B = 4(3x - 5)^2 - (3x - 5)(2x + 5) - (3x + 5)(3x - 5)$

$B = (3x - 5)[4(3x - 5) - (2x + 5) - (3x + 5)]$

$B = (3x - 5)[12x - 20 - 2x - 5 - 3x - 5]$

$B = (3x - 5)(7x - 30)$.

Chapter 7

Remarkable identities

How to Apply

Developments

For the exercises 1 to 5, it is required to develop each expression using the remarkable identities.

1 $A = (a + 2)^2$; $B = (3x + 5)^2$; $C = (x^2 + 3y)^2$; $D = (2{,}5\,x^2 + 4x)^2$.

2 $E = (a - 4)^2$; $F = (4x - 7)^2$; $G = (2x - 1{,}5)^2$; $H = (x^3 - 2xy)^2$.

3 $I = (a + 3)(a - 3)$; $J = (4x + 9)(4x - 9)$; $K = (x^2 - 2y)(x^2 + 2y)$;
$L = (x^2 - y^3)(x^2 + y^3)$.

4 $M = (-xy + 2y)^2$; $N = (-x^2 - 8)(-x^2 + 8)$; $O = (-y^2 - 10y)^2$.

5 $P = \left(2x + \frac{1}{4}\right)^2$; $Q = \left(\frac{x^2}{6} - \frac{3x}{2}\right)^2$; $R = \left(\frac{x}{4} + \frac{2}{3}\right)\left(\frac{x}{4} - \frac{2}{3}\right)$.

For the exercises 6 to 8, it is required to develop and reduce each algebraic expression.

6 $S = x + x(x + 2)^2$; $T = 2 - 2(x + 2)^2$; $U = (x - 2)(x + 2)^2$.

7 $V = (2 + 3x)^2 - 16x^2$; $W = -(-4y + 3)^2 + 4$; $X = 9 - (-3t - 2)^2$.

8 $Y = (3x + 4)^2 - (2x - 5)^2 - (5x - 2)(5x + 2)$; $Z = 2(-2x - 3)^2 - 3(-x + 4)^2$.

Chapter 7

Remarkable identities

How to Apply

Factorizations

For the exercises 9 to 12, it is required to factorize each expression using the remarkable identities.

9 $A = a^2 + 2a + 1$; $B = 4x^2 + 25 + 20x$; $C = 4xy + 4y^2 + x^2$.

10 $D = a^2 - 6a + 9$; $E = 16x^2 + 81 - 72x$; $F = -4x^2 + x^4 + 4$.

11 $G = a^2 - 16$; $H = 4x^2 - 49$; $I = 25x^2 - 64y^2$.

12 $J = \frac{a^2}{4} - a + 1$; $K = 16b^2 - \frac{25}{9}$; $L = \frac{2}{3}c + 1 + \frac{c^2}{9}$.

For the exercises13 to 15, it is required to write each expression in one of the form $k(a+b)^2$, $k(a-b)^2$ or $k(a+b)(a-b)$.

13 $M = 3a^2 + 12a + 12$; $N = 4x^3 - 12x^2 + 9x$; $O = 2x^3 - 8x$.

14 $P = -9x^2 + 30x - 25$; $Q = -16x^2 + 9$; $R = -4x^2 - 36x - 81$.

15 $S = 2x^3 + 8x^2y + 8xy^2$; $T = -x^4 + 12x^3 - 36x^2$; $U = -4x^3y + 9xy^3$.

For the exercises 16 and 17, it is required to factorize each « difference of two square ».

16 $V = (2 + 3x)^2 - 16x^2$; $W = 4 - (4y - 3)^2$; $X = 9 - (3t + 2)^2$.

17 $Y = (3x + 1)^2 - (2x - 5)^2$; $Z = (2x^2 + 5)^2 - (x^2 - 3)^2$.

Chapter 7

Remarkable identities

How to Apply

Numerical applications

18 Calculate each expression using the remarkable identities :

1º $a = (80 + 1)^2$; $b = (80 - 1)^2$; $c = (80 + 1)(80 - 1)$.

2º $d = 102^2$; $e = 98^2$; $f = 102 \times 98$.

19 Calculate the numerical expressions :

1º $g = 75^2 - 25^2$; $h = \left(\frac{13}{2}\right)^2 - \left(\frac{7}{2}\right)^2$.

2º $i = \left(\frac{2}{3}\right)^2 + 2 \times \frac{2}{3} \times \frac{1}{3} + \left(\frac{1}{3}\right)^2$; $j = \left(\frac{11}{4}\right)^2 - 2 \times \frac{11}{4} \times \frac{3}{4} + \left(\frac{3}{4}\right)^2$.

Know-how to be Competent

20 **Multiple choice questions** For each question choose the best answer with justification :

	Answer *A*	Answer *B*	Answer *C*
1º $4x^2 + x^4 + 4 = \dots$	$(2x + 2)^2$	$(2x + x^2)^2$	$(2 + x^2)^2$
2º $-a^2 - 4b^2 + 4ab = \dots$	$-(a + 2b)^2$	$-(a - 2b)^2$	$(-a - 2b)^2$
3º $(-3x + 6)^2 = \dots$	$9(x - 2)^2$	$-9(x - 2)^2$	$-3(x - 2)^2$
4º $-(2x + 3)(2x - 3) = \dots$	$-4x^2 - 9$	$-4x^2 + 9$	$4x^2 + 9$

21 Develop and reduce the algebraic expressions :

1º $P(x) = (x + 4)^2 \times (x - 4)^2$; $Q(x) = \left[36 - (x - 6)^2\right]^2$.

2º $R(x) = [6x - 2(x - 1)]^2 - [4x - (-x + 3)]^2$.

3º $S(x) = 7 - 2(2x + 1)^2 - (3 - 2x)(3 + 2x) + 3(1 - x)^2$.

22 For each algebraic expression, take a common factor, then factorize :

1º $A = (3x - 4)^2 + (4 - 3x)(x + 2) - 3x + 4$.

2º $B = (4x - 10)(3x - 1) - (4x^2 - 25) - (5 - 2x)^2$.

3º $C = (2x - 6)^2 - 3(x - 3)(3x + 1) - x^2 + 9$.

4º $D = (x + 3)(7x - 2) - (2 - 7x)(2x + 3) - 49x^2 + 28x - 4$.

Chapter 7

Remarkable identities

Know-how to be Competent

23 For every algebraic expression, perform convenient grouping, then factorize :

$E = 2x^3 + 3x^2 - 8x - 12$; $F = 4x^3 - 16x^2 - 9x + 36$.

24 Factorize the differences of two squares:

$G = 4(x-1)^2 - 9$; $H = 25 - 16(2x-1)^2$; $I = 25(x+1)^2 - 16(2x-1)^2$.

25 Write each of the following expressions in the form of a product of three factors :

$J = x^3 - 9x$; $K = x^4 - 81$; $L = 9(x-1)(x-2)^2 + 4(1-x)$.

26 Consider the algebraic expression $M = (xy+2)^2 + (x-2y)^2$.

1° Develop and reduce the expression M.

2° Deduce the factorization of the expression M.

27 Consider two numbers a and b such that : $a + b = 8$ and $a - b = 3.5$.
Without finding a and b, calculate the numerical value of each expression :

$N = a^3 - ab^2 + a^2 b - b^3$; $P = a^2 - (b-1)^2$

$Q = a(2b - 3a) - b(2a - 3b)$; $R = a^2(a+b) - 2ab(a+b) + b^2(a+b)$.